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STOCHASTIC SEISMIC RESPONSE OF MULTIPLY-SUPPORTED SECONDARY SYSTEMS IN FLEXIBLE-BASE STRUCTURES

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SUMMARY

A formulation has been proposed for the transfer function of a secondary system response while the primary system is supported on a compliant soil and the excitation comprises of translational ground motion at its base. For this purpose, the earlier formulation of the authors for the fixed-base case, which exactly considers the interaction between the two sub-systems and is based on the use of their individual modal properties, has been extended. Also, the concept of modifying the input excitation for the interaction accelerations (associated with the soil–structure interaction) has been used. An example P–S system and three example earthquake excitations have been considered to illustrate the proposed formulation and to estimate the expected response peak amplitudes in the secondary system. This study shows that 'detuning' of the tuned systems may occur in case of significant soil–structure interaction. Further, for the reasons of both safety and economy, ignoring the interaction effects in designing the secondary systems may not always be justified.

KEY WORDS: multiply-supported secondary system; flexible-base primary system; stochastic seismic response; PSDF-based approach; transfer function; fixed-base modes

1. INTRODUCTION

It is convenient to analyse the structures, like power plant installations and industrial structures, for ensuring the seismic safety of multiply-supported secondary systems, like piping and equipment, by directly synthesizing the individual modal properties of the primary and secondary systems. Various approaches proposed in the past two decades for this purpose include the response spectrum-based approaches (e.g., those by Sackman and Kelly,[1] Hernried and Sackman,[2] and Igusa and Der Kiureghian[3]) and the approaches based on the characterization of seismic hazard by ground Power Spectral Density Function (PSDF) (e.g. those by Lee and Penzien,[4] Muscolino,[5] Saudy *et al.*,[6] and Dey and Gupta[7]). Whereas the response spectrum-based approaches usually involve the calculation of the modal properties of the combined system

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by perturbation techniques, the PSDF-based approaches involve the formulation of the seismic response in frequency domain in terms of the (fixed-base) modal properties of the classically damped sub-systems. Several response spectrum approaches based on the use of floor response spectra, e.g. those by Asfura and Der Kiureghian,[8] Burdisso and Singh,[9] Saudy *et al.*,[10] have also been proposed for the response calculations.

All of the approaches mentioned above are based on the assumption that there is negligible effect of the relative motion of the foundation of the primary system (with respect to the surrounding soil) on the response of a secondary system. Unless a structure is built on the stiff soil or hard rock conditions, however, its seismic response may be affected by significant alterations in the frequencies and damping of the structure–foundation system. Due to the translation and rotation of the foundation relative to the soil, the values of the system frequencies are lowered as compared to those of the fixed-base structure, depending on the stiffness of the structure relative to the soil. This reduction in frequencies is accompanied by the dissipation of a considerable amount of the vibrational energy due to (i) the frequency-dependent radiation damping, and (ii) the frequency-independent material damping due to the internal friction. The resulting level of damping in the foundation is significantly different from the structure damping in most of the cases, and hence, the complete structure–foundation cannot be assumed to be classically damped. It is usual to analyse the structure–foundation system by treating the foundation and the structure as two separate units, with the interaction forces of equal magnitude acting in opposite directions on the two sub-systems. The force-deformation relationships and damping characteristics of the foundation are described by the complex-valued impedance functions. These functions depend on the frequency of excitation, and are obtained independently as functions of the properties of the layered soil medium and geometry and depth of embedment of the foundation by modelling the soil medium as a visco-elastic half-space (e.g. see References 11–13, etc.), or by using the finite element technique (e.g. see Reference 14).

Due to the frequency dependence of the impedance functions, it is convenient to analyse the linear structure–foundation systems in the frequency domain. Also, by transforming the equations of motion of the structure–foundation system by using the fixed-base modes of the structure, it is possible to circumvent the calculation of the complex damped mode shapes of the system. This approach was first proposed by Tajimi[15] and later used by Chopra and Gutierrez,[16] Luco,[17] Gupta and Trifunac,[18,19] Wu and Smith[20] in the SSI studies. It is also possible to account for the accelerations associated with the relative motion of foundation to the soil by scaling the Fourier amplitudes of the input motion at different frequencies.[19]

In this paper, the approach of Dey and Gupta[7] has been extended to study the secondary system response in case of the primary systems being founded on the compliant soil. The responses of the primary and secondary systems have been described in terms of their individual fixed-base mode shapes, and the approach of Gupta and Trifunac[19] has been used to formulate the transfer function of the desired response function of the secondary system. For this purpose, the free-field ground motion has been assumed to consist of the translational component only, and the foundation input motion has been assumed to be the same as the free-field translation. An example system has been considered to illustrate the proposed formulation by obtaining the transfer functions of several response functions. The proposed formulation has also been used to obtain the 'expected' largest peaks for some of these functions and to have a qualitative assessment of the SSI effects in case of different mass ratios, site conditions and ground motions.

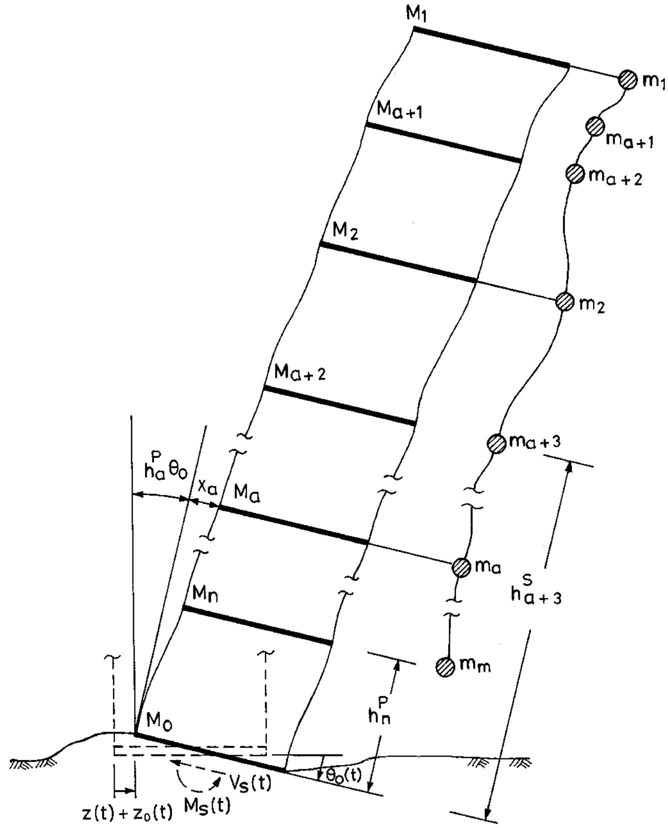

Figure 1. An idealized n-DOF primary-foundation system supporting a m-DOF secondary system

2. FORMULATION OF THE TRANSFER FUNCTION

2.1. 'Fixed-base' primary system response

Consider a linear, classically damped, n-Degree-of-Freedom (DOF) primary system supporting a linear, classically damped, m-DOF secondary system as shown in Figure 1. The first a DOFs of the primary system are assumed to be attached to the first a DOFs of the secondary system, through the elements with the stiffnesses, K_1, K_2, $\dots$, K_a, and dampings, C_1, C_2, $\dots$, C_a. Let $\{X(t)\}$ denote the displacements along the DOFs of the primary system relative to the foundation, and $\{u(t)\}$ denote the displacement vector for the secondary system. For the first a DOFs, the secondary system displacements are measured relative to the corresponding support DOFs of the primary system, while the remaining DOFs are measured relative to the foundation. The foundation input motion is assumed to be the same as the free-field ground translation, and thus the free-field rocking and the effects of kinematic interaction are neglected. Let the foundation undergo translation, $z_0(t)$, and rotation, $\theta_0(t)$, relative to the surrounding soil medium. Thus, by

using the substructure approach, the decoupled primary system may be considered as subjected to (i) the base excitations, $(\ddot{z}(t)+\ddot{z}_0(t))$ and $\ddot{\theta}_0(t)$, (ii) interaction forces, $f(t)$, acting at the attachment points to the secondary system, and (iii) to the interaction forces between the foundation and the half-space, i.e. to the base shear, $V_s(t)$ and base moment, $M_s(t)$. Accordingly, the equations of motion for the n DOFs of the primary system may be written as

$$[M]\{\ddot{X}(t)\}+[C]\{\dot{X}(t)\}+[K]\{X(t)\}=-[M]\{1\}(\ddot{z}(t)+\ddot{z}_0(t))-[M]\{h^{\mathrm{P}}\}\ddot{\theta}_0(t)+\{f(t)\} \tag{1}$$

where $[M]$, $[C]$ and $[K]$, respectively, represent the mass, damping and stiffness matrices of the primary system, and the elements of vector $\{h^{\mathrm{P}}\}$ $(=\{h_1^{\mathrm{P}}\ h_2^{\mathrm{P}}\dots h_n^{\mathrm{P}}\}^{\mathrm{T}})$ denote the heights of the masses, $M_1, M_2, \dots, M_n$ above the base. Since the primary and secondary systems are connected at a DOFs, the first a elements of $\{f(t)\}$ are the forces applied by the members connecting them, and the remaining $(n-a)$ elements are zeroes. The kth element of $\{f(t)\}$ is given by $(C_k\dot{u}_k(t)+K_k u_k(t))$ for $k=1,2,3,\dots,a$, where, C_k and K_k, respectively, are the damping and stiffness of the element connecting the secondary system to the primary system at the kth DOF, and $u_k(t)$ is the displacement for the kth secondary DOF.

Assuming the fixed-base primary system to be classically damped, its response can be expanded in terms of the (real-valued) orthonormal mode shapes of the system. Following this, we can express the displacement for the jth primary DOF in frequency domain as[7]

$$X_j(\omega)=\sum_{r=1}^{n}\phi_j^{(r)}H_r(\omega)\left(-\alpha_r\ddot{z}(\omega)+\sum_{k=1}^{a}\phi_k^{(r)}(\mathrm{i}\omega C_k+K_k)\,u_k(\omega)\right),\quad j=1,2,\dots,n \tag{2}$$

provided the interaction accelerations, $\ddot{z}_0(t)$ and $\ddot{\theta}_0(t)$, are negligibly small. In equation (2), $\phi_j^{(r)}$ denotes the jth element of the rth primary mode shape, $\{\phi^{(r)}\}$, α_r $(=\{\phi^{(r)}\}^{\mathrm{T}}[M]\{1\})$ is the modal participation factor in the rth mode,

$$H_r(\omega)=\frac{1}{\omega_r^2-\omega^2+2\mathrm{i}\zeta_r\omega_r\omega},\quad r=1,2,\dots,n \tag{3}$$

is the modal transfer function relating the displacement of the Single-Degree-of-Freedom (SDOF) oscillator in the rth (fixed-base) primary mode to the input ground acceleration, and $\ddot{z}(\omega)$ and $u_k(\omega)$, respectively, are the Fourier transforms of the corresponding time-dependent variables, $\ddot{z}(t)$ and $u_k(t)$. Further, in equation (3), ω_r and ζ_r, respectively, denote the natural frequency and damping ratio in the rth primary mode.

2.2. *Interaction accelerations*

When the SSI effects are significantly large, equation (2) needs to be modified to account for the contributions of interaction accelerations. To express these accelerations in terms of the input ground motion and interaction forces (as applied by the secondary system), we consider the equations of equilibrium of the complete primary structure–foundation system in translation and rotation. With the expansion of the primary system response in terms of the (fixed-base) primary modes, these equations can be written in frequency domain as

$$V_s(\omega)+\sum_{j=1}^{n}\sum_{r=1}^{n}M_j\phi_j^{(r)}\ddot{q}_r(\omega)+m_{\mathrm{T}}(\ddot{z}(\omega)+\ddot{z}_0(\omega))+m_{\mathrm{HT}}\ddot{\theta}_0(\omega)-\sum_{k=1}^{a}(\mathrm{i}\omega C_k+K_k)u_k(\omega)=0 \tag{4}$$

$$M_s(\omega) + \sum_{j=1}^{n} \sum_{r=1}^{n} M_j h_j^{\mathrm{P}} \phi_j^{(r)} \ddot{q}_r(\omega) + m_{\mathrm{HT}}(\ddot{z}(\omega) + \ddot{z}_0(\omega)) + I_{\mathrm{T}} \ddot{\theta}_0(\omega) - \sum_{k=1}^{a} (\mathrm{i}\omega C_k + K_k) u_k(\omega) h_k^{\mathrm{P}} = 0 \tag{5}$$

where $m_{\mathrm{T}} = \sum_{j=0}^{n} M_j$ is the total mass of the primary structure–foundation system, $m_{\mathrm{HT}} = \sum_{j=1}^{n} M_j h_j^{\mathrm{P}}$ is the moment of the entire primary structure–foundation system about the ground level, and $I_{\mathrm{T}} = I_0 + \sum_{j=1}^{n} (I_j + M_j h_j^{\mathrm{P}^2})$ is the moment of inertia of the primary structure–foundation system about a horizontal axis at the ground level. M_0 and I_0 represent the mass and mass moment of inertia of the foundation, and I_j represents the mass moment of inertia of the jth floor mass, M_j, about a horizontal axis through its mass centre. The terms, $V_s(\omega)$, $M_s(\omega)$, $\ddot{z}_0(\omega)$, and $\ddot{\theta}_0(\omega)$ are the Fourier transforms of $V_s(t)$, $M_s(t)$, $\ddot{z}_0(t)$, and $\ddot{\theta}_0(t)$, respectively. Further,

$$\ddot{q}_r(\omega) = \omega^2 H_r(\omega) \left(\alpha_r(\ddot{z}(\omega) + \ddot{z}_0(\omega)) + \gamma_r \ddot{\theta}_0(\omega) - \sum_{k=1}^{a} \phi_k^{(r)} (\mathrm{i}\omega C_k + K_k) u_k(\omega) \right), \quad r = 1, 2, \ldots, n \tag{6}$$

is the acceleration corresponding to the rth principal coordinate in frequency domain, as obtained from equation (1). In equation (6), $\gamma_r (= \{\phi^{(r)}\}^{\mathrm{T}} [M] \{h^{\mathrm{P}}\})$ is the modal participation factor in the rth mode for the base rocking. In writing equations (4) and (5), the motions have been assumed to be small and the contributions of the gravitational forces have been neglected.

The foundation is considered to be a rigid, massless slab of negligible thickness, which is attached to the surface of the underlying soil medium through linear springs and viscous dashpots. The properties of these springs and dashpots are represented by the complex-valued impedance functions, $K_{\mathrm{VV}}(\omega)$, $K_{\mathrm{VM}}(\omega)\ (= K_{\mathrm{MV}}(\omega))$ and $K_{\mathrm{MM}}(\omega)$. These impedance functions relate the interaction forces, $V_s(\omega)$ and $M_s(\omega)$, between the foundation and the underlying soil with the foundation displacements as

$$\begin{Bmatrix} V_s(\omega) \\ M_s(\omega)/L \end{Bmatrix} = \begin{bmatrix} K_{\mathrm{VV}} & K_{\mathrm{VM}} \\ K_{\mathrm{MV}} & K_{\mathrm{MM}} \end{bmatrix} \begin{Bmatrix} z_0(\omega) \\ L\theta_0(\omega) \end{Bmatrix} \tag{7}$$

Here L denotes a suitable length of reference of the foundation. For example, it is taken as the radius of a circular foundation of equal area in case of a rectangular slab foundation. The impedance functions are assumed to be obtained independently for the given foundation–soil system, e.g. for a surface or embedded foundation resting on visco-elastic uniform or layered half-space, or on layered soil deposit over a rigid half-space (e.g. see References 11–14).

Substitution of equations (6) and (7) in equations (4) and (5) and solving the simultaneous equations in $\ddot{z}_0(\omega)$ and $\ddot{\theta}_0(\omega)$ leads to

$$\ddot{z}_0(\omega) = \chi_{zz}^{(1)}(\omega) \ddot{z}(\omega) + \sum_{k=1}^{a} (\mathrm{i}\omega C_k + K_k) u_k(\omega) \chi_{(zz)k}^{(2)}(\omega) \tag{8}$$

$$\ddot{\theta}_0(\omega) = \chi_{\theta z}^{(1)}(\omega) \ddot{z}(\omega) + \sum_{k=1}^{a} (\mathrm{i}\omega C_k + K_k) u_k(\omega) \chi_{(\theta z)k}^{(2)}(\omega) \tag{9}$$

Here $\chi_{zz}^{(1)}(\omega)$ and $\chi_{\theta z}^{(1)}(\omega)$, respectively, represent the transfer functions relating the interaction accelerations, $\ddot{z}_0(\omega)$ and $\ddot{\theta}_0(\omega)$, to the input ground acceleration, $\ddot{z}(\omega)$, as also obtained by Gupta and Trifunac.[19] Similarly, the terms, $\chi_{(zz)k}^{(2)}(\omega)$ and $\chi_{(\theta z)k}^{(2)}(\omega)$ denote the transfer functions relating the interaction accelerations to the interaction force, $(\mathrm{i}\omega C_k + K_k) u_k(\omega)$, acting at the kth attachment point of the secondary system. The expressions for these transfer functions are given in the

appendix. It may be observed that when the effects of SSI are negligibly small, the values of $\chi^{(1)}_{zz}(\omega)$, $\chi^{(1)}_{\theta z}(\omega)$, $\chi^{(2)}_{(zz)k}(\omega)$ and $\chi^{(2)}_{(\theta z)k}(\omega)$ tend to become zero and then, the interaction accelerations can be neglected.

2.3. 'Flexible-base' primary system response

On including the contributions of $\ddot{z}_0(\omega)$ and $\ddot{\theta}_0(\omega)$ as in equations (8) and (9), equation (2) may be modified to

$$X_j(\omega) = \sum_{r=1}^{n} \phi_j^{(r)} \left(\ddot{z}(\omega) R_r(\omega) + \sum_{k=1}^{a} S_k^{(r)}(\omega)(\mathrm{i}\omega C_k + K_k) u_k(\omega) \right), \quad j = 1, 2, \ldots, n \tag{10}$$

with

$$R_r(\omega) = H_r(\omega)(-\alpha_r(\chi^{(1)}_{zz}(\omega) + 1) - \gamma_r \chi^{(1)}_{\theta z}(\omega)) \tag{11}$$

and

$$S_k^{(r)}(\omega) = H_r(\omega)(-\alpha_r \chi^{(2)}_{(zz)k}(\omega) - \gamma_r \chi^{(2)}_{(\theta z)k}(\omega) + \phi_k^{(r)}) \tag{12}$$

2.4. Secondary system response without SSI effects

The equations of motion for the decoupled secondary system may be written as

$$[m]\{\ddot{u}(t)\} + [c]\{\dot{u}(t)\} + [k]\{u(t)\} = \{F(t)\} \tag{13}$$

where, $[m]$, $[c]$ and $[k]$, respectively, are the mass, damping and stiffness matrices of the fixed-base secondary system. $\{F(t)\}$ is the vector of input excitation to the secondary system due to the motions at the far ends of the connecting members, and is given by

$$\{F(t)\} = -[\tilde{m}]\{\ddot{\tilde{X}}(t)\} - [\tilde{c}]\{\dot{\tilde{X}}(t)\} - [\tilde{k}]\{\tilde{X}(t)\} - [m]\{1\}\ddot{z}(t) - [m]\{1\}\ddot{z}_0(t) - [m]\{h^{\mathrm{S}}\}\ddot{\theta}_0(t) \tag{14}$$

where $\{h^{\mathrm{S}}\}$ $(= \{h_1^{\mathrm{S}}\ h_2^{\mathrm{S}} \ldots h_m^{\mathrm{S}}\}^{\mathrm{T}})$ is the vector whose elements represent the heights of the masses, $m_1, m_2, \ldots, m_m$ of the secondary system above the base. Further, $[\tilde{m}]$, $[\tilde{c}]$ and $[\tilde{k}]$ are the matrices of $m \times a$ dimension each. The element, (i, j), of $[\tilde{m}]$ is equal to the element, (i, j), of $[m]$ for $i = j$ and is equal to zero otherwise. The matrices, $[\tilde{c}]$ and $[\tilde{k}]$, are obtained by retaining the first a columns of the matrices, $[c]$ and $[k]$, respectively, and by considering the dampings and stiffnesses of the members connecting the primary and secondary systems, i.e. $C_1, C_2, \ldots, C_a$, and $K_1, K_2, \ldots, K_a$, to be zero. Further, the vectors, $\{\ddot{\tilde{X}}(t)\}$, $\{\dot{\tilde{X}}(t)\}$ and $\{\tilde{X}(t)\}$, constitute the first a elements of $\{\ddot{X}(t)\}$, $\{\dot{X}(t)\}$ and $\{X(t)\}$, respectively.

Assuming the fixed-base secondary system to be classically damped, its response can be expanded in terms of the (real-valued) orthonormal mode shapes of the system. For negligible interaction accelerations, this leads to the following expression of the displacement for the pth secondary DOF in frequency domain (see Reference 7 for details)

$$u_p(\omega) = \sum_{l=1}^{m} \psi_p^{(l)} h_l(\omega) \left(-\sum_{i=1}^{m} \psi_i^{(l)} \left(m_{ii}\ddot{z}(\omega) + \sum_{j=1}^{a} A_{ij}(\omega) X_j(\omega) \right) \right), \quad p = 1, 2, \ldots, m. \tag{15}$$

Here m_{ii} is the ith diagonal element of $[m]$; $\psi_i^{(l)}$ represents the ith element of the lth (orthonormal) modal vector, $\{\psi^{(l)}\}$, of the fixed-base secondary system

$$h_l(\omega) = \frac{1}{\Omega_l^2 - \omega^2 + 2\mathrm{i}\xi_l\Omega_l\omega}, \quad l = 1, 2, \ldots, m \tag{16}$$

is the modal transfer function relating the displacement of the lth SDOF system oscillator to the input ground acceleration; and

$$A_{ij}(\omega) = \omega^2\tilde{m}_{ij} - \mathrm{i}\omega\tilde{c}_{ij} - \tilde{k}_{ij} \tag{17}$$

denotes the transfer function between the ith element of the input excitation, $F(t)$, and the displacement, $X_j(t)$, along the jth primary DOF. In equation (16), Ω_l and ξ_l, respectively, denote the natural frequency and damping ratio for the lth mode of the (fixed-base) secondary system. Further, in equation (17), $\tilde{m}_{ij}$, $\tilde{c}_{ij}$ and $\tilde{k}_{ij}$, respectively, denote the (i, j)th elements of the matrices, $[\tilde{m}]$, $[\tilde{c}]$ and $[\tilde{k}]$.

2.5. Secondary system response with SSI effects

On adding the terms, $\ddot{z}_0(\omega)$ and $h_i^{\mathrm{S}}\ddot{\theta}_0(\omega)$, to $\ddot{z}(\omega)$ for the case of significant SSI effects, and on substitution of equations (8)–(10), equation (15) may be expressed as

$$\begin{aligned} u_p(\omega) - \sum_{k=1}^{a}\Bigg[&\sum_{l=1}^{m}\sum_{i=1}^{m}\psi_p^{(l)}\psi_i^{(l)}h_l(\omega)(\mathrm{i}\omega C_k + K_k) \\ &\times\Bigg(-m_{ii}(\chi_{(zz)k}^{(2)}(\omega) + h_i^{\mathrm{S}}\chi_{(\theta z)k}^{(2)}(\omega)) + \sum_{j=1}^{a}\sum_{r=1}^{n}A_{ij}(\omega)\phi_j^{(r)}S_k^{(r)}(\omega)\Bigg)\Bigg]u_k(\omega) \\ = W_p(\omega)\ddot{z}(\omega), &\quad p = 1, 2, \ldots, m \end{aligned} \tag{18}$$

with

$$\begin{aligned} W_p(\omega) = \sum_{l=1}^{m}\sum_{i=1}^{m}\psi_p^{(l)}\psi_i^{(l)}h_l(\omega)\Bigg(&-m_{ii}(1 + \chi_{zz}^{(1)}(\omega) + h_i^{\mathrm{S}}\chi_{\theta z}^{(1)}(\omega)) \\ &+ \sum_{j=1}^{a}\sum_{r=1}^{n}A_{ij}(\omega)\phi_j^{(r)}R_r(\omega)\Bigg), \quad p = 1, 2, \ldots, m. \end{aligned} \tag{19}$$

The m linear, simultaneous equations, as in equation (18), completely describe the m unknown secondary system displacements at a given frequency, ω. Due to the linear dependence of these displacements on the input ground motion, $\ddot{z}(\omega)$, the solution of simultaneous equations will give the m transfer functions, each relating the displacement along a DOF of the secondary system to the input ground acceleration. These transfer functions can then be used in equation (10), if desired, to obtain the transfer functions for the primary system displacements, $X_1(t), X_2(t), \ldots, X_n(t)$. Since the other response quantities would be related linearly to the primary and secondary system displacements, their transfer functions can also be easily determined from the transfer functions of these displacements (see Reference 7 for details).

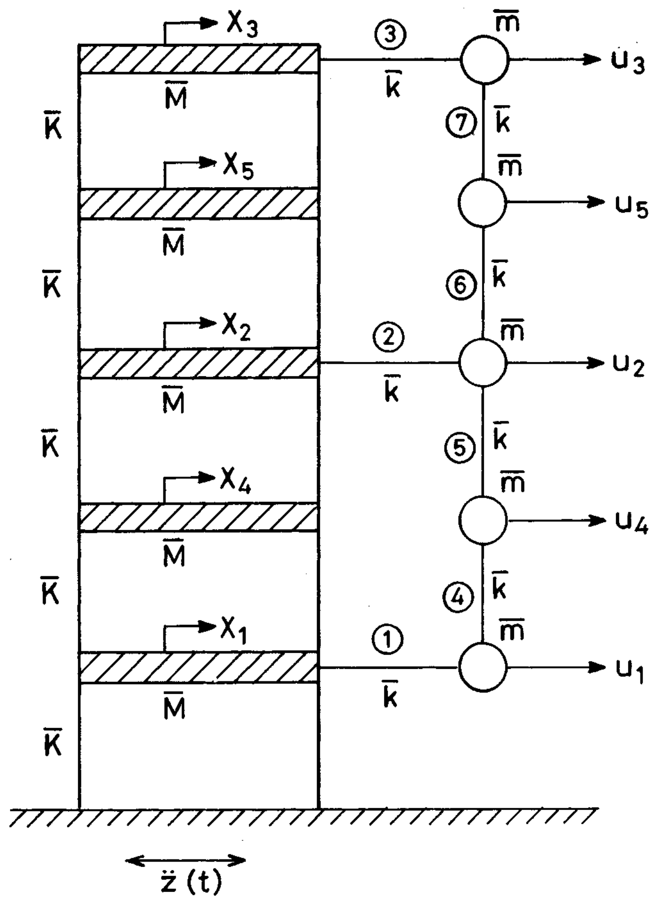

Figure 2. Idealized model of example P–S system

3. NUMERICAL EXAMPLES

3.1. Transfer functions

The proposed formulation for the transfer functions of the secondary system responses has been illustrated by considering the example P–S system shown in Figure 2. This consists of a 5-DOF primary system and a 5-DOF secondary system with three attachment points, with the vertical and rotational degrees of freedom assumed to be of little significance. For the primary system, uniform floor masses and interstory stiffnesses have been considered with $\bar{M} = 1{\cdot}0 \times 10^5$ kg and $\bar{K} = 2{\cdot}0 \times 10^4$ kN/m. The mass and stiffness properties of the secondary systems have also been considered to be uniform with the ratio, $\bar{k}/\bar{m}$, kept at $31{\cdot}22\ \mathrm{sec}^{-2}$. The (fixed-base) natural frequencies of both primary and secondary systems in the example system are 4·03, 11·75, 18·52, 23·79, 27·14 and 4·02, 5·59, 8·49, 9·68, 11·41 rad/sec, respectively. Since the fundamental frequencies of the decoupled primary and secondary systems are same and the

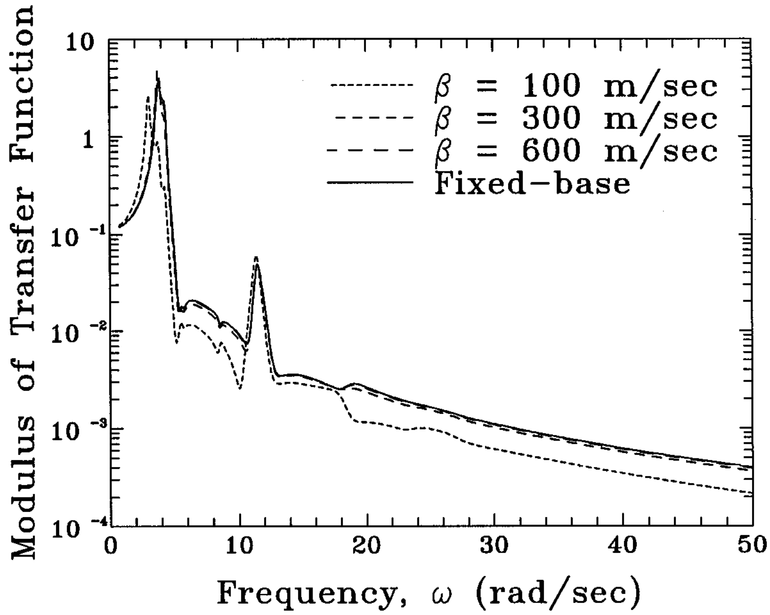

Figure 3. Modulii of transfer functions for $(u_2 + X_2)$ response in case of different soil conditions

second frequency of the primary system is very close to the fifth frequency of the secondary system, this corresponds to a tuned P–S system. The damping ratio has been assumed to be 0·05 for all the primary modes and 0·02 for all the secondary modes. Further, the secondary system here has been assumed to be moderately heavy with the mass ratio, $m/M = 0{\cdot}02$.

The foundation is considered to consist of a square slab with length of reference, $L = 3{\cdot}0$ m and of negligible mass as compared to the floor masses. The slab is assumed to be attached to the surface of a uniformly visco-elastic half-space with the impedance functions as obtained by Wong and Luco.[12] The soil is thus characterized by its mass density, ρ, Poisson's ratio, v, shear modulus of rigidity, G, shear wave velocity, $\beta = \sqrt{G/\rho}$, and by the hysteretic damping ratio, ς. The impedance functions are directly proportional to G and L, and depend on v, ς and the dimensionless frequency, $\bar{\omega} = \omega L/\beta$. Let the mass density, ρ, hysteretic damping ratio, ς, and Poisson's ratio, v, be taken as equal to 1000 kg/m^3, 0·02 and 0·3, respectively. Further, the mass moments of inertia of the floor masses, i.e. I_j's, are assumed to be negligible.

The proposed formulation has been used to obtain the transfer functions for various response quantities of the secondary system. The displacement transfer functions for the $(X_2 + u_2)$ and u_5 responses (i.e. the displacements for the second and fifth DOFs as measured relative to the foundation) have been presented in Figures 3 and 4. The force transfer function for the member 2 (connecting the primary and secondary systems) has been shown in Figure 5. In each figure, transfer functions for three different shear wave velocities, $\beta = 100$, 300 and 600 m/sec have been compared with those corresponding to the fixed-base condition (as formulated by Dey and Gupta[7]). As expected, it is observed in each figure that for $\beta = 100$ m/sec, the peak corresponding to the fundamental modes of the primary and secondary systems has been significantly shifted to the left with a significant reduction in its amplitude. Also, as the shear wave velocity increases, the

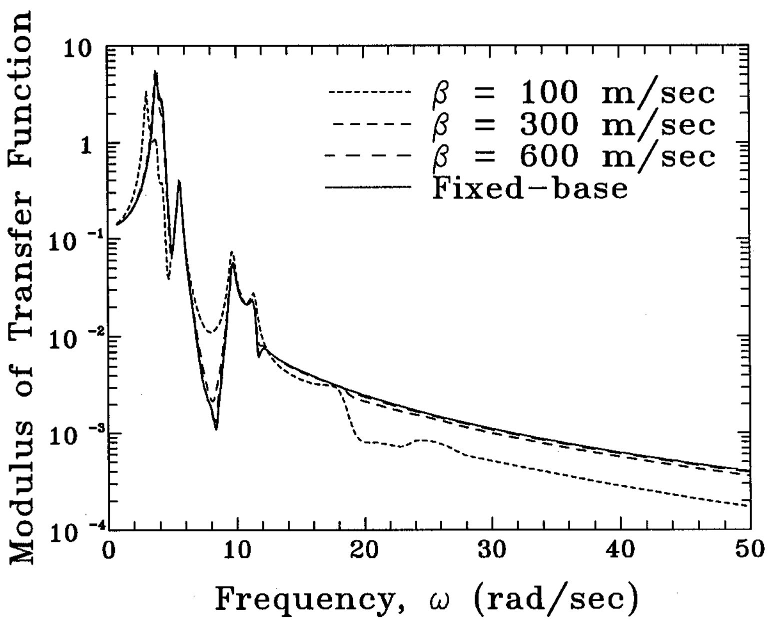

Figure 4. Modulii of transfer functions for u_5 response in case of different soil conditions

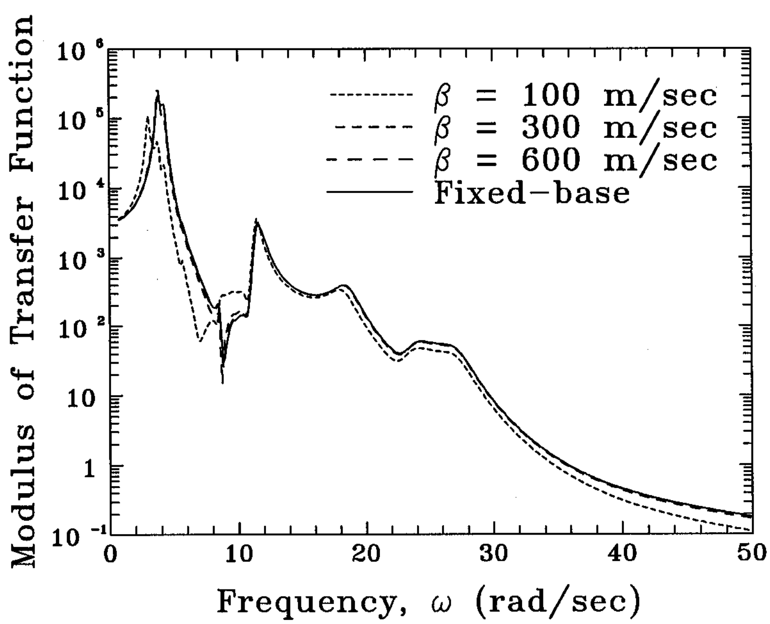

Figure 5. Modulii of transfer functions for force in member 2 in case of different soil conditions

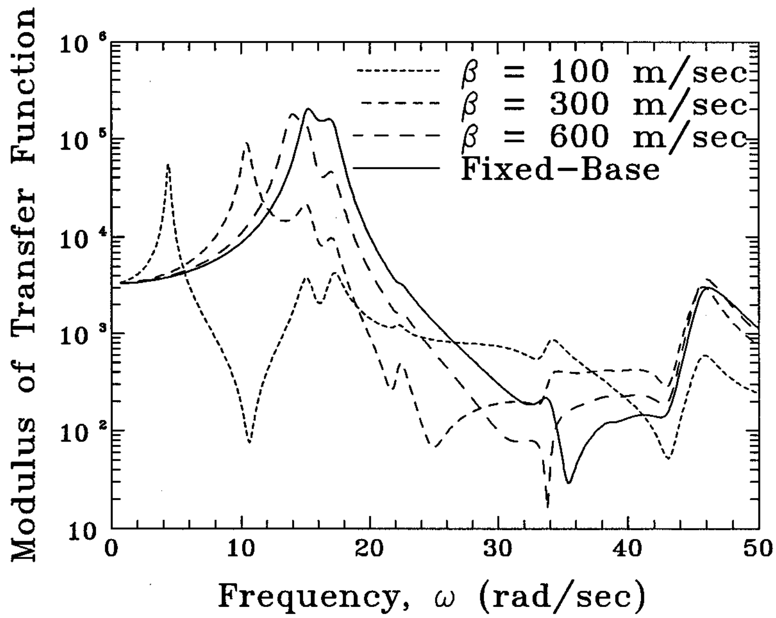

Figure 6. Modulii of transfer functions for force in member 2 of extra-stiffened P–S system

transfer function curves approach that for the fixed-base model. The effects of SSI are not noticeable in the transfer function curves for $\beta = 600$ m/sec. It may be mentioned that a stiffer example system would have been associated with greater effects of SSI for the same value of β. To illustrate this, the results of Figure 5 have been obtained again with the values of $\bar{K}$ and $\bar{k}$ taken sixteen times larger (and thus, with the natural frequencies of the two sub-systems being four times larger). As shown in Figure 6, these results clearly indicate that the fundamental frequency of the combined P–S system is reduced by about 30 per cent and the corresponding peak is reduced by about 55 per cent in case of $\beta = 300$ m/sec. These reductions in the case of $\beta = 100$ m/sec are even greater, while for $\beta = 600$ m/sec, those are not insignificant. Thus, in case of the 'white-noise' like excitations, SSI may lead to reduced responses in secondary systems for softer soils, and such reductions may be substantially larger for stiffer P–S systems, with the other parameters like foundation size remaining unchanged. It is interesting to observe in Figure 6 that as the shear velocity decreases, the peaks corresponding to the fundamental frequencies of the two sub-systems merge into a single peak. This means that in the tuned systems, the effect of tuning on the combined system frequencies decreases with the increasing effects of SSI. This follows from a reduced level of tuning between the secondary system and a more flexible primary system (due to SSI), even though the fixed-base primary system is perfectly tuned with the secondary system. Extending this argument, it is also possible to have a situation in which a non-tuned P–S system becomes a tuned system due to SSI, provided that the primary system is founded on a very soft soil and that the primary system is stiffer than the secondary system.

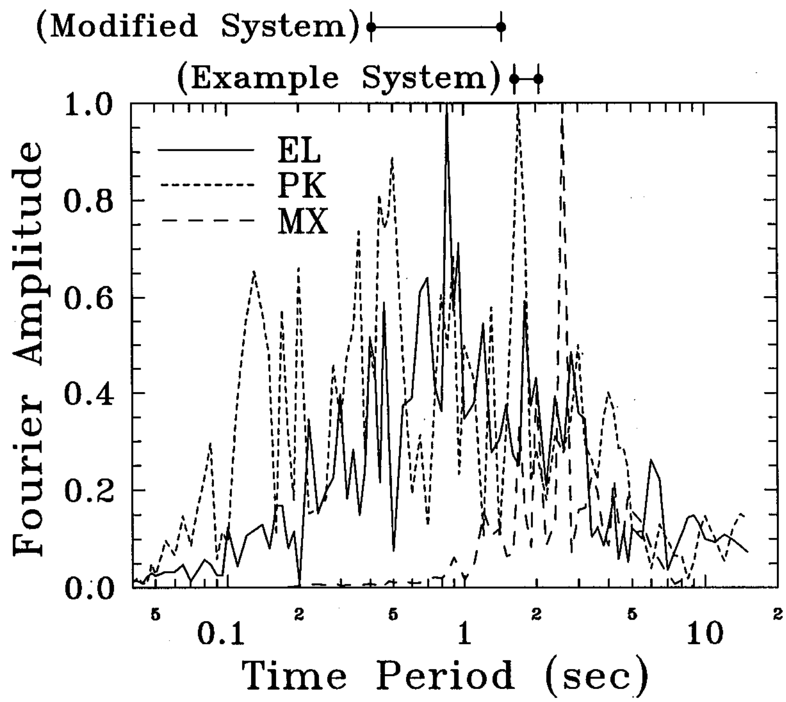

Figure 7. Comparison of the normalized Fourier spectra of the El Centro (EL), Mexico City (MX) and Parkfield (PK) ground motions

3.2. Stochastic response

The proposed formulation for the transfer functions can be used to obtain the stochastic estimates of the peak responses by considering a PSDF-based approach. In this paper, we consider the same approach as used by Dey and Gupta.[7] Three different example ground motions have been considered to characterize the excitation process, $\ddot{z}(t)$. These motions are: (i) recorded S00E component of Imperial Valley earthquake of 18 May 1940 at El Centro site, (ii) synthetic accelerogram for the horizontal component of Michoacan earthquake, 1985 at Mexico City site (as in Reference 21), and (iii) recorded vertical component of Parkfield earthquake of 27 June 1966 at Cholame, Shandon site. The Fourier spectra of these ground motions, as normalized to the respective maximum amplitudes, have been compared in Figure 7.

It may be observed that compared to the El Centro motion, the Parkfield motion has greater domination of high-frequency waves while the Mexico City motion is dominated by longer period waves. For each example motion, response spectra for 2, 5 and 10 per cent damping ratios have been generated, and the spectrum-compatible PSDFs corresponding to these response spectra have been obtained by assuming the strong motion duration to be 30 sec. From these PSDFs for each accelerogram, an 'envelope PSDF' has been obtained to characterize the ground excitation. For relating the r.m.s. and the peak values of the response functions, the peak factors based on the order statistics formulation, as in References 22–24, have been used.

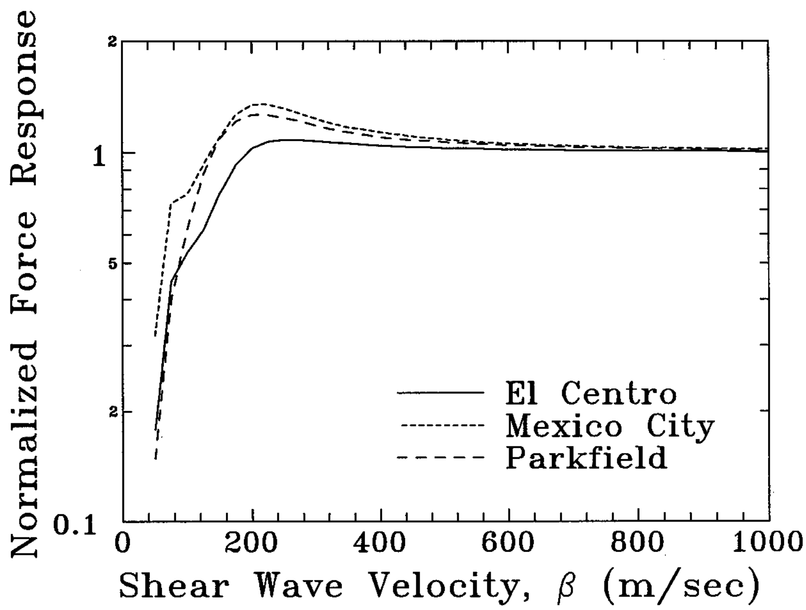

Figure 8. Variation of normalized force response in member 2 with β for different excitations

The example system has been analysed for various shear wave velocity, β, values ranging from 50 m/sec for very soft soil conditions to 1000 m/sec for the stiff soil conditions. Figure 8 shows the comparison of the 'expected' peak force values in the member 2 for the three example excitations. Each curve in the figure shows the normalized response with respect to that for the fixed-base system. Thus, more this normalized response deviates from 1.0, greater are the effects of SSI. Due to the logarithmic scale used to represent the normalized responses, the magnitude of this deviation however appears same, irrespective of its sign. As expected, the normalized response approaches unity with the increasing value of β. Except for a mild rise around $\beta = 200$ m/sec, all the three curves show a steep decline for the soft soils. For $\beta = 50$ m/sec, the response may get reduced by as much as 85 per cent. This steep decline is primarily due to the larger structure–soil damping and hence smaller fundamental frequency peak in the transfer function. The mild rise as seen in the figure is associated with greater energy available in the excitation around the fundamental period of the structure–soil system. In fact, since the excitation for the Mexico City site shows a narrow-band concentration of energy around a period of 2.5 sec, this mild rise is greater in case of this particular excitation as the fundamental period shifts towards 2.5 sec with the softening of the soil (see the range of the fundamental periods of the example system in Figure 7 as the shear wave velocity is varied between 100 m/sec and infinity). To further appreciate the combined effects of excitation energy available around the fundamental period and the increased system damping due to SSI, the results shown in Figure 8 are obtained again by using the transfer function modulii of Figure 6 (for an extra-stiffened P–S system), instead of those in Figure 5, and are shown in Figure 9. Here, the system is particularly very stiff to the Mexico City motion and the fundamental period increases substantially along with much increased dissipation of energy for the softer soils (again see the range of fundame ntal periods of the modified system in Figure 7).

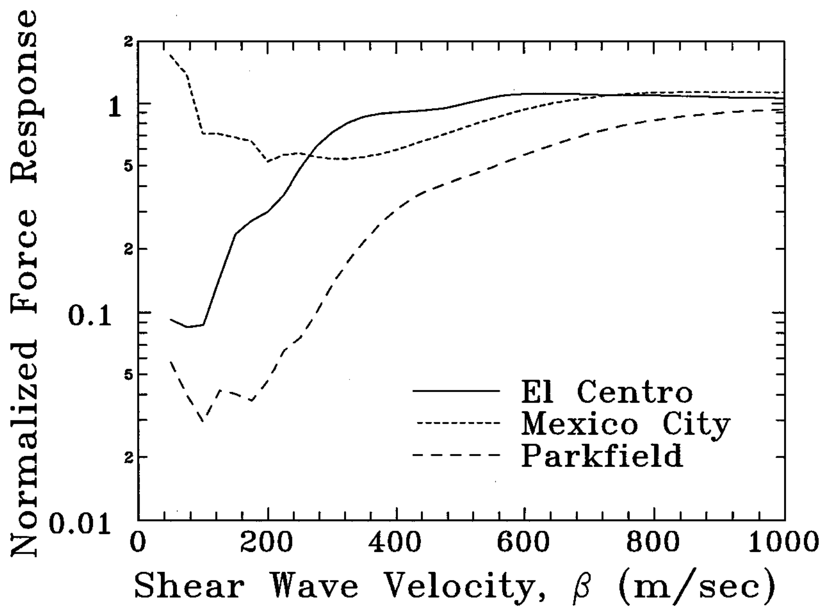

Figure 9. Effect of earthquake ground motion on normalized force response in member 2 of extra-stiffened P–S system

Due to this, the Mexico City curve exhibits the net effect of the opposing trends, with the curve rising for very soft soils after declining for the medium soft soils. The other two curves show the continuous reductions in the responses with the decreasing value of β, as expected, and the response may be reduced by as much as 97 per cent for $\beta = 100$ m/sec. It is clear from these results that even though the precise nature and extent of SSI effects on response may depend on the energy available in the ground excitation around the system period and on the relative stiffness of the superstructure to the soil, the SSI effects may cause the response of secondary systems to be reduced substantially for the very soft soils and significantly (say, around 50 per cent) for the medium soft soils. It may sometimes lead to larger responses also as shown in Figure 10. In this figure, results for four different systems derived from the example system have been shown in case of the El Centro ground motion. In Systems A and B, the $\bar{K}$ value is 16 times that in the example system. The value of $\bar{k}$ is same in System A and 16 times in System B as compared to that in the example system. In Systems C and D, the $\bar{K}$ value is 64 times that in the example system. The value of $\bar{k}$ is sixteen times in System C and 64 times in System D as compared to that in the example system. The mass values are kept same as in the example system with $m/M = 0{\cdot}02$. Table I shows the fundamental periods of the primary and secondary systems in these (derived) systems. It may be noted that all the four systems are much stiffer than the example system with the System D being the most stiff. Figure 10 shows that Systems C and D are associated with some SSI effects on response even at $\beta = 1000$ m/sec. In System A, the force may increase to about 4.5 times that in the fixed-base case for $\beta = 100$ m/sec, while in System B, this increase may be up to 6.5 times for $\beta = 400$ m/sec. Thus, the effects of SSI are not always on the conservative side, and those may take place at 'not so soft' soils as well. Figure 10 also shows that for the same

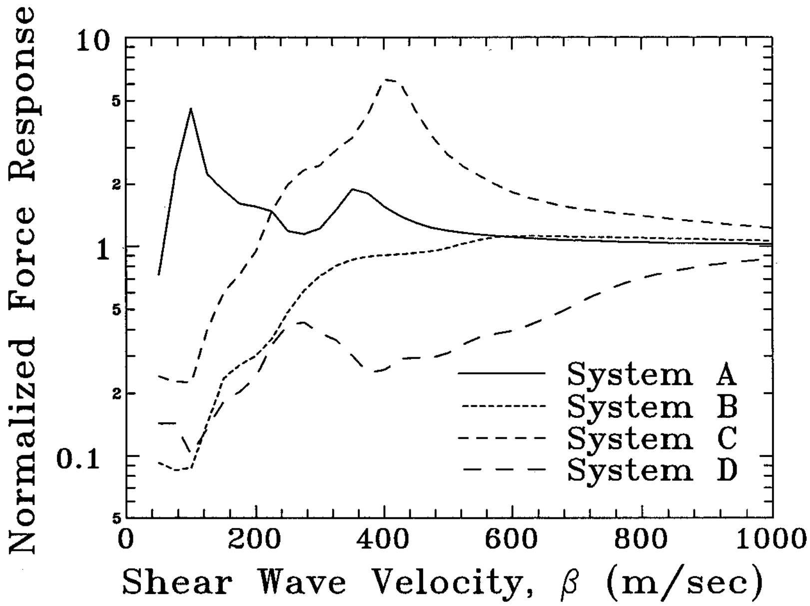

Figure 10. Variation of normalized force response in member 2 with β for different derived systems

Table I. Fundamental periods of derived systems

Derived system	Fundamental period (sec)	
	Primary system	Secondary system
A	0·39	1·56
B	0·39	0·39
C	0·19	0·39
D	0·19	0·19

ground excitation, very different effects of SSI are obtained with changing β, primarily due to the frequencies of the combined P–S system.

The above results were obtained for $m/M = 0{\cdot}02$. To see the effects of this ratio on the effects of SSI, results have also been obtained for $m/M = 0{\cdot}2$, 0·002 and 0·0002 representing very heavy to very light secondary system cases. The natural frequencies of the sub-systems have been kept unchanged, as in the example system, and the results have been obtained again for the El Centro ground motion. As shown in Figure 11, those have been compared with the results for $m/M = 0{\cdot}02$. It may be observed that except for the case of $m/M = 0{\cdot}2$ representing significant interaction between the primary and secondary systems, other cases with moderate to little interaction show a gradual decrease in the normalized response with the decrease in shear wave velocity. This decrease in normalized response is primarily due to the increased system damping with increasing SSI effects as discussed earlier. In case of significant interaction, however, it is well known that the tuned modes of the primary and secondary systems show up as two closely spaced

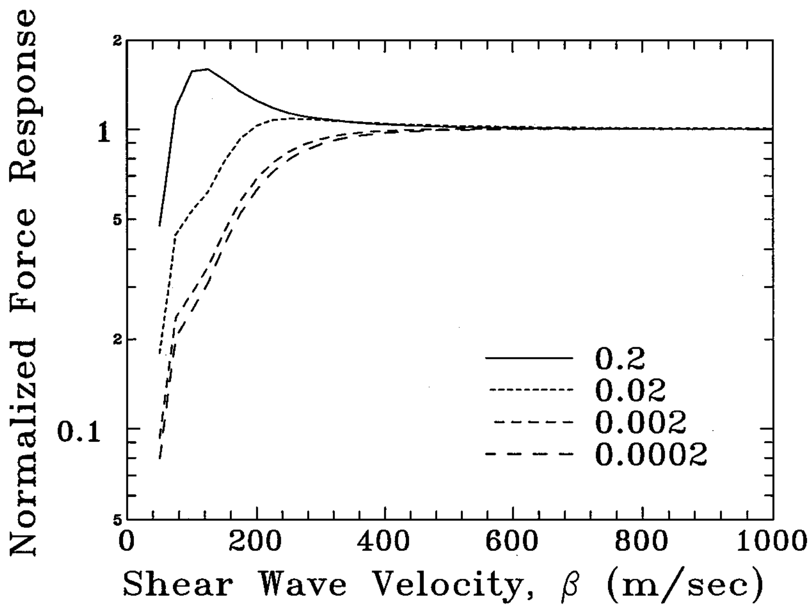

Figure 11. Variation of normalized force response in member 2 with β for different m/M values

peaks of reduced amplitudes (as compared to the cases of moderate to little interaction) in the transfer functions of various response functions. When the SSI effects become significant at low shear wave velocities, the effects of tuning decrease, as discussed in the previous sub-section, and the phenomenon of closely spaced peaks of smaller amplitudes, as observed in the fixed-base case, does not take place any longer. As a result, the transfer function peak corresponding to the fundamental frequency actually becomes higher than that in the fixed-base case. This is the reason why the normalized response is seen to rise at lower β values for $m/M = 0{\cdot}2$. Below $\beta = 100$ m/sec, however, the response starts falling off, as the effects of increased damping begin to dominate the effects of 'detuning'.

It may be mentioned that the above observations are not likely to change qualitatively, if the impedance functions for more complicated foundation geometries are used in place of those considered here (i.e. for the surface foundations supported on the visco-elastic, homogeneous half-space). Those may however not be applicable to soft soils in the near source regions, due to the observed strong non-linear behaviour of such soils.[25] Moreover, the non-linear soil response may also lead to large differential motions, leading to significantly higher stresses in the first storey columns of spatially extended primary systems.[26] Further, it is assumed in this illustrative study that the ground motions with different energy distributions can be recorded at a site, irrespective of the local site conditions. This is not true, since the local site conditions strongly influence the Fourier spectrum shape of the recorded motion, e.g. the Mexico City type of motion is characteristic of the very soft soil deposits. Considering more realistic range of β (instead of the wide range as considered) for a given excitation is however unlikely to qualitatively change the above observations, since the three example excitations together cover a broad range of energy distributions (see Figure 7).

4. CONCLUSIONS

A formulation has been presented for the transfer function of the seismic response of a secondary system, while the primary system is supported on a compliant soil. This formulation is convenient to use in any PSDF-based stochastic approach, as it only requires the knowledge of the (fixed-base) modal properties of the primary and secondary systems and the impedance functions of the foundation. Also, this exactly accounts for the interaction between the primary and secondary sub-systems, besides that between the structure and soil. This formulation may be easily used in those cases also where the foundation has a complex geometry and is embedded in a multi-layered soil medium and/or more advanced models have been used to calculate the dynamic properties of the primary and secondary systems.

The proposed approach has been illustrated by considering an example P–S system and obtaining the transfer functions for a few displacement and force response functions for different SSI levels. The observed trends conform to the known effects of SSI and to the importance of the relative stiffness of the structure to the soil in these effects. It has been found that the tuned P–S systems may sometimes become detuned in case of significant SSI. Also, an illustrative study with three different ground motions has shown that the 'expected' peak response estimates may not always be on the conservative side in the case of significant soil–structure interaction. In fact, depending upon the type of excitation and relative stiffness of the structure to the soil, these estimates may exceed several times those based on the fixed-base assumption even in the case of 'not so soft' soils. Hence, unless the soil conditions are known to be very stiff at the site under consideration, the effects of SSI should not be ignored in the design of secondary systems.

APPENDIX

Transfer functions, χ's

The transfer functions for the interaction acceleration as in equations (8) and (9) may be expressed as

$$\chi_{zz}^{(1)}(\omega) = \frac{\omega^2}{\Delta}(L(Lm_{\mathrm{T}}(\omega)K_{\mathrm{MM}} - m_{\mathrm{HT}}(\omega)K_{\mathrm{VM}}) + \omega^2(m_{\mathrm{HT}}^2(\omega) - m_{\mathrm{T}}(\omega)I_{\mathrm{T}}(\omega))) \tag{20}$$

$$\chi_{(zz)k}^{(2)}(\omega) = \frac{\omega^2}{\Delta}(G_k(\omega)(\omega^2 I_{\mathrm{T}}(\omega) - L^2K_{\mathrm{MM}}) + P_k(\omega)(LK_{\mathrm{VM}} - \omega^2 m_{\mathrm{HT}}(\omega))) \tag{21}$$

$$\chi_{\theta z}^{(1)}(\omega) = \frac{\omega^2}{\Delta}(m_{\mathrm{HT}}(\omega)K_{\mathrm{VV}} - Lm_{\mathrm{T}}(\omega)K_{\mathrm{VM}}) \tag{22}$$

$$\chi_{(\theta z)k}^{(2)}(\omega) = \frac{\omega^2}{\Delta}(G_k(\omega)(K_{\mathrm{VM}} - \omega^2 m_{\mathrm{HT}}(\omega)) + P_k(\omega)(\omega^2 m_{\mathrm{T}}(\omega) - K_{\mathrm{VV}})) \tag{23}$$

with

$$\Delta = L^2(K_{\mathrm{VV}}K_{\mathrm{MM}} - K_{\mathrm{VM}}^2) + \omega^2(2Lm_{\mathrm{HT}}(\omega)K_{\mathrm{VM}} - I_{\mathrm{T}}(\omega)K_{\mathrm{VV}} - L^2 m_{\mathrm{T}}(\omega)K_{\mathrm{MM}}) + \omega^4(m_{\mathrm{T}}(\omega)I_{\mathrm{T}}(\omega) - m_{\mathrm{HT}}^2(\omega)), \tag{24}$$

$$G_k(\omega) = 1 + \omega^2 \sum_{r=1}^{n} \alpha_r H_r(\omega) \phi_k^{(r)} \tag{25}$$

and

$$P_k(\omega) = h_k^{\mathrm{P}} + \omega^2 \sum_{r=1}^{n} \gamma_r H_r(\omega) \phi_k^{(r)} \tag{26}$$

In equations (20)–(24),

$$m_{\mathrm{T}}(\omega) = m_{\mathrm{T}} + \omega^2 \sum_{r=1}^{n} M'_r H_r(\omega) \tag{27}$$

$$m_{\mathrm{HT}}(\omega) = m_{\mathrm{HT}} + \omega^2 \sum_{r=1}^{n} Z'_r H_r(\omega) \tag{28}$$

and

$$I_{\mathrm{T}}(\omega) = I_{\mathrm{T}} + \omega^2 \sum_{r=1}^{n} I'_r H_r(\omega) \tag{29}$$

respectively, denote the total mass, moment of total mass (about the ground level) and moment of inertia of the complete system of masses of the primary structure–foundation system, as modified for the flexibility in the superstructure.[19] Further, $M'_r = \sum_{j=1}^{n} M_j \phi_j^{(r)} \alpha_r$, $Z'_r = \sum_{j=1}^{n} M_j \phi_j^{(r)} \gamma_r$, and $I'_r = \sum_{j=1}^{n} M_j \phi_j^{(r)} \gamma_r h_j^{\mathrm{P}}$ represent the properties of the primary system in the rth fixed-base mode.

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